Math 601 - Advanced Combinatorics I - Fall 2020
Zoom meeting room: 142-012-467[1]

Basic Information

Instructor: Maria Gillespie, Maria.Gillespie@colostate.edu
Office: 125 Weber Building/Zoom meeting room 142-012-467 (See footnote for password.)

Course web page: See the Canvas page for most announcements. Homeworks and links to videos will also be posted at:
http://mathematicalgemstones.com/maria/Math601Fall20.php
for interested students who are auditing the class.

Class time and location: MWF 1:00 pm, either outdoors near Weber Hall (whenever weather permits) or on Zoom meeting room: 142-012-467 (See footnote for password.)

Office hours: Wednesday 12:30-1:00 PM (outdoors or zoom based on weather) and Thursday 9:00-10:00 AM (only zoom), or send me an email or a Zoom message at any time.

Books: There are **no required books**; coming to lecture (or catching up via recorded videos) will be sufficient for learning all material. Some helpful references are:

- *Representation Theory*, by Fulton and Harris
- *Introduction to Quantum Groups and Crystal Bases*, by Hong and Kang
- *Crystal Bases*, by Bump and Schilling
- *The Symmetric Group*, by Bruce Sagan
- *Combinatorics of Coxeter Groups* by Björner and Brenti.

Prerequisites: Familiarity with the basics of group theory and symmetric functions is helpful.

[1]Password is the number of distinct ways to rearrange the numbers in the Zoom ID.

Course policies

The grading for this class will be entirely homework based. Some homeworks will be long and cumulative in place of exams.

Homework will be posted every Friday and will be due the following Fridayby the end of the day. Please submit your homework as a single typed or scanned PDF file. There will be two types of homeworks: **ordinary weekly homeworks** and **cumulative homeworks.** Which type you will get on each week will vary (see the schedule below).

- **Ordinary weekly homework format:** Each homework problem will be assigned a number of points based on difficulty. A 1 or 2 point problem is a routine exercise. A 3, 4, or 5 point problem is harder and may involve some clever thinking. Problems worth 6 points or over are very hard, and problems worth 10 or more are either open problems or recently published results. (Partial credit will be given for strong insights towards a problem.)

  The points you earn are cumulative, and each homework is graded out of a maximum of 10 points. Your score on the homework will be

  $$\min(T, 10)$$

  where T is the total number of points of the problems you handed in correct solutions to. *Make sure you clearly indicate which problems you are handing in and what their point values are!*

  You may hand in a set of problems whose total score is greater than 10 **if and only if** removing any one of the problems will make the total less than 10.

  Collaboration is permitted, but you must list all coauthors on a problem's solution at the top of the page.

- **Cumulative homework format:** These should be thought of as small take-home tests. Collaboration is not permitted, though you may talk with your peers about other homework problems from the weekly homeworks in order to gain more understanding while working on the cumulative homework. These 'midterm' homeworks will be worth 30 points each. All problems on these homeworks must be solved to receive full credit.

Late homework policy: Please hand in homework on time, and if some emergency comes up just ask for an extension. However, Cumulative Homework C must be handed in by December 4 and Homework 11 must be handed in by Dec 11 for them to count.

Grading: Your lowest homework grade (calculated as a percentage, including cumulative homeworks) will be dropped to account for possible emergencies or illness. Otherwise your final grade is the weighted average of all your homework grades.

Tentative Schedule

- Week 1: (Aug 24, 26, 28) Introduction: combinatorics in representation theory; groups, Lie groups, Lie algebras, the basic combinatorial objects (Young tableaux, crystals, root systems, Dynkin diagrams)
- Week 2: (Aug 31, Sep 2, 4) Lie groups vs Lie algebras and representation correspondence; the exponential map; examples of Lie groups and where they arise
  - Homework 1 due Sep 4
- Week 3: (Sep 9, 11) Representation theory of $\mathfrak{sl}_2$
  - Homework 2 due Sep 11
- Week 4: (Sep 14, 16, 18) Tensor product diagrams, ballot words in two letters
  - Homework 3 due Sep 18
- Week 5: (Sep 21, 23, 25) Representation theory of $\mathfrak{sl}_3$, weight lattice
  - **Cumulative Homework A due Sep 25**
- Week 6: (Sep 28, 30, Oct 2) $\mathfrak{sl}_3$ - tensor products, tableaux crystals, Schur function characters
  - Homework 4 due Oct 2
- Week 7: (Oct 5, 7, 9) Representations of $\mathfrak{sl}_n$, Stembridge axioms for type A, general LR rule
  - Homework 5 due Oct 9
- Week 8: (Oct 12, 14, 16) Classification of semisimple Lie algebras by root systems, dynkin diagrams
  - Homework 6 due Oct 16
- Week 9: (Oct 19, 21, 23) General crystal base theory
  - **Cumulative Homework B due Oct 23**
- Week 10: (Oct 26, 28, 30) Weyl groups and Coxeter groups in other Lie types
  - Homework 7 due Oct 2
- Week 11: (Nov 2, 4, 6) Representations of Weyl groups, Schur-Weyl duality (and example of why it doesn't work in type B)
  - Homework 8 due Nov 6
- Week 12: (Nov 9, 11, 13) Representations of S_n

  - Homework 9 due Nov 13

- Week 13: (Nov 16, 18, 20) Frobenius map and Littlewood-Richardson rule
  - Homework 10 due Nov 20
- Week 14: (Nov 30, Dec 2, 4) Projective reps of S_n, Q-schur functions
  - **Cumulative Homework C due Dec 4**
- Week 15: (Dec 7, 9, 11) Q-schur function crystal mystery
  - Homework 11 due Dec 11