Math 567: Abstract Algebra I
Homework 9

10 points total. Due Friday, Apr 1 by 1:10 pm in class.

Problems

1. (1 point) We know that π is transcendental (over $\mathbb{Q}$). Using this, prove that $\pi + \sqrt{2}$ is transcendental.

2. (2 points) In class, we proved that any degree 2 extension of a field not of characteristic 2 is generated by a square root. Find, with proof, an example of a degree 2 extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$ (which does have characteristic 2) that is not generated by a square root over $\mathbb{F}_2$.

3. (3 points - half a point per part) Artin ch 15 exercise 3.4.

4. (2 points - half a point per part) Artin ch 15 exercise 4.2.

5. (1 point each) Artin ch 15 exercises 3.8 and 3.10.

Bonus Problem

(+1 point:) Towards proving that e and π are transcendental, first prove that e is irrational as follows. Recall the power series expansion

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n.$$

Setting $x = 1$ we have $e = \sum_{n=0}^{\infty} \frac{1}{n!}$. Show that if $e = p/q$ for integers p, q, multiplying both sides by $q!$ yields a tail of the summation that is less than $1/q$ and is therefore not an integer.