Math 567: Abstract Algebra I
Homework 8

10 points total. Due Friday, Mar 25 by 1:10 pm in class.

Problems

1. (1 point) The textbook claims in section 15.1 that any subfield of $\mathbb{C}$ contains $\mathbb{Q}$. Prove this.

2. (2 points) The fact that $\mathbb{Q}[\sqrt{2}] = \mathbb{Q}(\sqrt{2})$ means that one can "rationalize the denominator" of any fraction of the form $\frac{a+b\sqrt{2}}{c+d\sqrt{2}}$. Indeed, we know how to do this from previous algebra classes; simply multiply the top and bottom by $c - d\sqrt{2}$. Now, we also know that $\mathbb{Q}[\sqrt[3]{2}] = \mathbb{Q}(\sqrt[3]{2})$. How would you go about rationalizing denominators in this field?

3. (1 point) Artin Ch 15 problem 1.1.

4. (2 points each) Artin Ch 15 problems 2.1, 3.1, 3.3

Bonus Problem

(+1 point:) Artin Ch 15 problem 2.2.