Math 567: Abstract Algebra I
Homework 7

10 points total. Due Friday, Mar 11 by 11:10 am in class.

Problems

1. (3 points) The **restriction** of a representation V of G to a subgroup H of G is the same vector space V with the action of H inherited from the action of G. Is the restriction of an irreducible representation to a subgroup necessarily irreducible? If so, explain why. If not, give a counterexample.

2. (2 points) Let V be a representation of G and let V^* be its dual, as defined in class. Prove that $\chi_{V^*}(g) = \chi_V(g^{-1})$ for all $g \in G$.

3. (1 point each) Artin Chapter 10 problems 4.3(a), 4.3(d), 4.8, 6.1, 6.2.

Bonus Problem

(+1 point:) Suppose H is a subgroup of G and V is a representation of H, thought of as a $\mathbb{C}H$-module. Then the **induced representation** of V from H to G is defined as

$$\mathrm{Ind}_H^G V := \mathbb{C}G \otimes V$$

where in the tensor product above, we are thinking of both $\mathbb{C}G$ and V as $\mathbb{C}H$-modules, and then intepreting the result as a $\mathbb{C}G$-module by the action on the left factor of the tensor product.

Compute the induced representation of the trivial representation of S_2 to S_3. Is it irreducible?