Math 567: Abstract Algebra I
Homework 5

10 points total. Due Friday, Feb 25 by 11:10 am in class.

Problems

1. (3 points) Consider the quotient ring $R_2 = \mathbb{C}[x,y]/(x+y, xy)$. Notice that S_2 acts on $\mathbb{C}[x,y]$ by the transposition (12) switching x with y; for instance, if we apply (12) to the polynomial $f(x,y) = 3x^2+2y$, we would get $3y^2 + 2x$.

   Since the ideal we are quotienting by is invariant under this S_2 action, it is a $\mathbb{C}S_2$-submodule, and therefore S_2 also acts on the quotient. Show that this quotient module R_2 decomposes as a direct sum of two irreducible S_2-modules (or equivalently as representations).

2. (2 points each) Artin chapter 10 problems 2.1, 2.3(a), 2.3(b). (Note, in 2.3 the group G stands for S_3.)

Bonus Problem

(+1 point:) Consider the quotient ring $R_3 = \mathbb{C}[x,y,z]/(x+y+z, xy+yz+zx, xyz)$ as an S_3-module by the permutation action that permutes the three variables x, y, z. Decompose this representation into irreducibles.