Math 567: Abstract Algebra I
Homework 2

10 points total. Due Friday, Feb 4 by 11:10 am in class.

Problems

1. (2 points) Prove that, if M is an R-module and N is a submodule of M, then M/N is an R-module.
2. (3 points) Suppose I, J are ideals of R. Prove that, as R-modules, we have $R/I \otimes R/J \cong R/(I + J)$.
3. (1 point) Use the above result to compute $\mathbb{Z}/n\mathbb{Z} \otimes \mathbb{Z}/m\mathbb{Z}$ (as $\mathbb{Z}$-modules) in terms of n and m.
4. (2 points) Artin Chapter 14 problem 2.2.
5. (1 point each) Artin Chapter 14 problems 2.4(a), 2.4(b).

Bonus Problem

(+1 point:) Suppose S is an R-algebra, and let M be any R-module. Show that $S \otimes_R M$ can be thought of as an S-module in a natural way. (This is often called *base change*, since we are in some sense changing M from an R-module into an S-module).