Math 566: Abstract Algebra I
Homework 7

10 points total. Due Friday, October 8 by 11:10 am in class.

Problems

1. (1 point) Prove that, for any vector spaces U, V, W over a field $\mathbb{F}$, we have

$$(U \oplus V) \otimes W \cong (U \otimes W) \oplus (V \otimes W).$$

   Show also that $\oplus$ and $\otimes$ are commutative operations on vector spaces (up to isomorphism).

2. (2 points) It is known that all of the irreducible representations of an abelian group are one-dimensional. Find all of the irreducible representations of the cyclic group C_5.

3. (2 points) Compute the dimension of the space $\mathbb{R}[x, y, z]^{(d)}$ of degree-d homogeneous polynomials in three variables x, y, z. Express your answer in terms of d.

4. (1 point each) Chapter 4 exercises 1.2, 1.3, 2.1, 3.1, Chapter 5 exercise 1.1.

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)

Chapter 4 exercises 1.1, 1.5, 2.3, 3.3.

Bonus

+1 onto your total homework score for correctly answering the following question:

Decompose the permutation representation of S_3 into a direct sum of irreducible representations.