Math 566: Abstract Algebra I
Homework 6

10 points total. Due Friday, October 1 by 11:10 am in class.

Problems

1. (1 point) What day of the week will it be 2^{100} days from today (the day this homework is due)? Use modular arithmetic to arrive at your answer.
2. (1 point) Prove that the group $\mathrm{GL}(V)$ of automorphisms of the vector space V is a group.
3. (1 point) Prove that $\mathrm{GL}(\mathbb{C}^n) \cong \mathrm{GL}_n(\mathbb{C})$.
4. (2 points) Prove that the set $\mathbb{C}[x,y]^{(d)}$ of all *homogeneous* polynomials of degree d in two variables is a vector space over $\mathbb{C}$ (with usual polynomial addition and scaling). Find a basis and compute its dimension in terms of d. (Note: a homogeneous polynomial of degree d is one in which the total degree of every monomial is d. For instance, $4xy^2 + y^3 - x^2y$ is homogeneous of degree 3, but $4xy^2 + 2x$ is not homogeneous.)
5. (2 points) Let V and W be vector spaces with bases $\mathcal{B}_v$ and $\mathcal{B}_W$ respectively. Show that a bijection $f : \mathcal{B}_v \to \mathcal{B}_w$ uniquely determines a linear transformation $\phi_f : V \to W$ (compatible with f on the basis elements), and that this transformation is an isomorphism. Conclude that there is only one vector space of dimension n up to isomorphism.
6. (1 point) Show that $\mathbb{R}[x]$ is isomorphic to the vector space $\mathbb{R}[x]_{\geq 1}$ of polynomials with no constant term.
7. (1 point each) Chapter 3 exercises 3.1 and 5.1.

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)
Chapter 3 exercises

Bonus

+1 onto your total homework score for correctly answering the following question:
Use the bonus question from homework 5 to show that if $n \neq 6$, S_n has only inner automorphisms.