

Math 566: Abstract Algebra I

Homework 5

10 points total. Due Friday, September 24 by 11:10 am in class.

Problems

- (2 points) Write down one representative for each of the conjugacy classes of S_5 . How many are there? How many elements does each conjugacy class have?
- (3 points) Let C_λ be the conjugacy class in S_n with *cycle type* $\lambda_1, \dots, \lambda_k$ (that is, the class of permutations having cycle lengths $\lambda_1, \dots, \lambda_k$). For each positive integer i , let m_i be the number of times i appears among $\lambda_1, \dots, \lambda_k$. Show that

$$|C_\lambda| = \frac{n!}{\prod_i i^{m_i} \cdot m_i!}.$$

- (1 point) Are the groups S_3 and $C_3 \times C_2$ isomorphic? Why or why not? What about C_6 and $C_3 \times C_2$? What about $C_3 \times C_2$ and $C_2 \times C_3$?
- (1 point) Is there an embedding of C_3 into S_3 such that $S_3/C_3 \cong C_2$? If so, write out the corresponding short exact sequence. Is there an embedding of C_2 into S_3 such that $S_3/C_2 \cong C_3$? If so, write out the corresponding short exact sequence.
- (1 point each) Chapter 2 exercises 8.4, 11.5, 12.4.

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)
Chapter 2 exercises 7.6, 8.1, 8.9, 9.1, 9.3, 9.4, 9.7, 11.1, 11.3, 11.5, 12.1, 12.2, M4.

Bonus

+1 onto your total homework score for correctly answering *all* of the questions below.

- Consider the conjugacy classes in S_n whose elements have order two.
 - Describe these conjugacy classes and the number of elements in them.
 - Show that for any fixed $n \neq 6$, none of these conjugacy classes of order 2 have the same size as the conjugacy class of transpositions.
 - Show that for $n = 6$, one of these conjugacy classes of order 2 does have the same size as the conjugacy class of transpositions.
 - Conclude that for $n \neq 6$, any automorphism of S_n has to permute the transpositions.

This is one step towards showing that only S_6 has an outer automorphism. More steps on future homeworks!