Math 566: Abstract Algebra I

Homework 4

10 points total. Due Friday, September 17 by 11:10 am in class.

Problems

1. (2 points) Let $\pi \in S_n$ be a permutation, and let $p \in S_n$ be another permutation written in cycle notation as
$$(p_{11}\, p_{12}\, \cdots\, p_{1i_1})\,(p_{21}\, p_{22}\, \cdots\, p_{2i_2})\,\cdots\,(p_{r1}\,\cdots\, p_{ri_r})$$
Prove that $\pi p \pi^{-1}$, written in cycle notation, is
$$(\pi(p_{11})\, \pi(p_{12})\, \cdots\, \pi(p_{1i_1}))\,(\pi(p_{21})\, \pi(p_{22})\, \cdots\, \pi(p_{2i_2}))\,\cdots\,(\pi(p_{r1})\,\cdots\, \pi(p_{ri_r})),$$
i.e., the permutation formed by applying π to each of the entries in the expansion in cycle notation.

2. (3 points) Define the product group $\mathbb{Z} \times \mathbb{Z}$ to be the set of ordered pairs (m, n) where m and n are both integers, along with vector addition as its operation (so $(m, n) + (r, s) = (m + r, n + s)$). This can be thought of as a lattice grid in the plane, and we will show in class that this is a group. Compute $|\mathrm{Aut}(\mathbb{Z} \times \mathbb{Z})|$.

3. (1 point each) Chapter 2 exercises 5.1, 5.4, 6.1, 6.4, 6.9.

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)

Chapter 2 exercises 5.3, 5.6, 6.5, 6.6, 6.7, 6.9.