Math 566: Abstract Algebra I
Homework 2

10 points total. Due Friday, September 3 by 11:10 am in class.

Problems

1. (3 points) Prove that every permutation can be expressed as a product of length-two cycles (called *transpositions*).
2. (3 points) Let π be a fixed permutation. Prove that the number of transpositions needed to express a permutation π as a product (as in the previous problem) is always either even or odd. If it is even, we say the *sign* of π is 1, written $\mathrm{sgn}(\pi) = 1$, and if it is odd, we say $\mathrm{sgn}(\pi) = -1$. (Hint: Prove that if $\mathrm{inv}(\pi)$ is even, then any product of transpositions must have an even number of them, and if it is odd then the number of transpositions is odd.)
3. (1 point) Prove that matrix multiplication is associative, by directly expanding out the coefficients using the formula for matrix multiplication.
4. (1 point each.) Exercises 2.2, 2.5, 2.6 in chapter 2 of the textbook.

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)
Chapter 2, exercises 1.1-1.3, 2.1, 2.3

Bonus problem

(+1 extra point if solved correctly) Prove that every group of even order contains an element of order 2.