Math 566: Abstract Algebra I
Homework 14

10 points total. Due Friday, Dec 10 by 11:10 am in class.

Problems

1. (1 point) Do "division with remainder" of $5 + 6i$ by $1 + i$ in the Euclidean domain $\mathbb{Z}[i]$ in two different ways. That is, express $5 + 6i$ as $q(1 + i) + r$ for two different pairs of complex numbers q, r for which $|r|^2 < |1 + i|^2$.

2. (2 points) Chapter 12 exercise 2.10. (Hint: First prove it is a PID. If you have already solved the Bonus on Homework 12 correctly then you may use your previous work; otherwise, try to do that problem first.)

3. (1 point each) Chapter 12 exercises 2.6(a), 2.6(b), 2.7, 3.2, 4.1(a), 4.1(b), 4.4

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)

Chapter 12 exercises 2.8, 4.10.

Bonus

(+1 point): Let G be a group and $\mathbb{F}$ any field. The **group ring** $\mathbb{F}G$ is the set of all formal linear combinations of elements of G with coefficients in $\mathbb{F}$. For instance, an element of $\mathbb{C}S_3$ might look like:

$$0.2 \cdot (\mathbf{12}) \, - \, (3 + i) \cdot (\mathbf{132})$$

where the permutations are shown in boldface to distinguish them from the complex number coefficients.

We can put a ring structure on the group ring by the usual addition on formal linear combinations, and multiplication $*$ given by the left and right distributive laws along with the rule

$$(a \cdot g) * (b \cdot h) = ab \cdot (gh)$$

where $a, b \in \mathbb{F}$ and $g, h \in G$.

Show that the group ring is in general a (possibly noncommutative) ring with identity, and show that it is a commutative ring if and only if G is abelian.