Math 566: Abstract Algebra I
Homework 12

10 points total. Due Friday, Nov 19 by 11:10 am in class.

Problems

1. (3 points) A *noncommutative ring* is a set R with an addition and multiplication operator that satisfies all of the ring axioms except for commutativity of multiplication. Let $\mathrm{Mat}_n(\mathbb{F})$ be the set of all $n \times n$ matrices with entries in a field $\mathbb{F}$. Prove that $\mathrm{Mat}_n(\mathbb{F})$ is a noncommutative ring with respect to matrix addition and multiplication. For which numbers n and fields $\mathbb{F}$ is it a commutative ring?

2. (2 points) The *characteristic* of a ring R is the smallest positive integer n such that adding 1_R to itself exactly n times results in 0_R. (If no such n exists we say the ring has infinite characteristic). Prove that if a field has finite characteristic n then n is prime, and give an example of a ring with non-prime finite characteristic.

3. (1 point each) Chapter 11 exercises 3.8, 4.1, 4.4, 7.2, 8.3.

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)

Chapter 11 exercises 4.2, 5.5, 6.2, 6.8, 7.1, 8.1.

Bonus

(+1 point): Chapter 11 exercise 3.10.