Math 566: Abstract Algebra I
Homework 11

10 points total. Due Friday, Nov 12 by 11:10 am in class.

Problems

1. (1 point) Let p be a prime. Using the multiplicative group of the field $\mathbb{Z}/p\mathbb{Z}$, prove that for any integer a, we have $a^p \equiv a \pmod{p}$. (This is called 'Fermat's Little Theorem'.)

2. (1 point) For which pairs of integers n and m is there a ring homomorphism from $\mathbb{Z}/n\mathbb{Z}$ to $\mathbb{Z}/m\mathbb{Z}$?

3. (3 points) Let R be any ring. Prove that the ring of formal power series $R[[x]]$ defined in class is indeed a ring.

4. (5 points - one point per part) Chapter 11 exercise 3.3

Recommended practice exercises

(DO NOT hand these in - these are just extra problems I recommend you look at if you'd like more practice.)

Chapter 11 exercises 1.8, 2.1, 3.6, 3.7, 3.10.

Bonus

(+1 point): (Constructing the outer automorphism of S_6). Recall on previous bonus problems we showed that only S_6 can have an outer automorphism. Show that there is a homomorphism $f : S_6 \to S_6$ defined on the generators $s_1, \ldots, s_5$ (where $s_i = (i\ i+1)$ as in the previous homework) as follows:

$$\begin{aligned}
f(s_1) &= (12)(34)(56),\\
f(s_2) &= (13)(25)(46),\\
f(s_3) &= (15)(26)(34),\\
f(s_4) &= (13)(24)(56),\\
f(s_5) &= (16)(25)(34)
\end{aligned}$$

by showing that the images $f(s_i)$ satisfy the braid relations and commutation relations that define the symmetric group. Then prove that f is an isomorphism, and conclude that it is an outer automorphism of S_6.