Math 502: Combinatorics
Homework 8

Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for scoring details.

Problems

1. (1) [1 point] Suppose G acts on X. Prove that the stabilizer of a point $x \in X$ is a subgroup of G.

2. (1+) [2 points] Sagan chapter 6 problem 8(a)

3. (2-) [3 points] Sagan chapter 6 problem 8(b)

4. (1+) [2 points] Sagan chapter 6 problem 9(a)

5. (1+) [2 points] Sagan chapter 6 problem 9(b)

6. (2-) [3 points] Sagan chapter 6 problem 10

7. (2-) [3 points] Show that $\mathrm{Frob}(\mathrm{Ind}_{S_2}^{S_3})V_{(1,1)} = e_{2,1}$ where $V_{1,1}$ is the sign representation of S_2.

8. (2+) [4 points] Show that
$$V_{1^n} \otimes_{\mathrm{inn}} V_\lambda = V_{\lambda^T}$$

where λ^T denotes the conjugate or transpose partition. Conclude that, under the Frobenius map, taking an inner product with the sign representation corresponds to applying the ω involution on symmetric functions.