Math 502: Combinatorics
Homework 2

Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for scoring details.

Problems

1. Evaluate the following using the properties of the Hall inner product discussed in class:
   - (1) [1 point] $\langle s_{(2,1)}, h_{1,1,1}\rangle$
   - (1) [1 point] $\langle s_{(3,1,1)}, s_{(3,2)}\rangle$
   - (1) [1 point] $\langle e_{(2,1)}, h_{(2,1)}\rangle$
   - (1) [1 point] $\langle p_{(3,2,2,1)}, p_{(3,2,2,1)}\rangle$
2. (1+) [2 points] Apply the ω involution to both sides of the Jacobi-Trudi formula to derive a formula for Schur functions in terms of elementary symmetric functions.
3. (2-) [3 points] (If you have NOT done this problem on the last homework, you may do it on this one!) Prove that
$$\sum_{\lambda} m_\lambda(x_1, x_2, \ldots)e_\lambda(y_1, y_2, \ldots) = \prod_{i,j=1}^{\infty}(1 + x_i y_j).$$
4. (2-) [3 points] Prove that
$$\sum_{\lambda} m_\lambda(x_1, x_2, \ldots)h_\lambda(y_1, y_2, \ldots) = \prod_{i,j=1}^{\infty}\frac{1}{1 - x_i y_j}.$$
5. (1) [1 point] Write out the six permutations of $1, 2, 3$ and the pairs of standard Young tableaux corresponding to each under the RSK bijection.
6. (1+) [2 points] Write out all rearrangments of the letters $1, 1, 2, 3$, and the pairs (S, T) of a semistandard and standard Young tableau, respectively, corresponding to each word under the RSK bijection.
7. (1+) [1 point] What two-line array corresponds to the following pair of semistandard Young tableaux under the RSK bijection?

4				
2	2	3		
1	1	1	3	4

,

3				
2	2	3		
1	1	1	1	2

8. (2-) [3 points] Show that the pairs (T, T) where T is a standard Young tableau correspond, under the RSK bijection, to the permutations that are involutions.
9. (2) [3 points] Using the fact that $s_\lambda = \sum_\mu K_{\lambda\mu} m_\mu$ and the properties of the Hall inner product, show that
$$h_\mu = \sum_\lambda K_{\lambda\mu} s_\lambda.$$