Math 502 - Combinatorics II
Spring 2020

Basic Information

Instructor: Maria Gillespie, Maria.Gillespie@colostate.edu
Office: 125 Weber Building

Course web page: http://mathematicalgemstones.com/maria/Math502Spring20.php
See also the Canvas course page.

Class time and location: MWF 10:00 am, Zoom meeting room: 142-012-467

Office hours: Monday 9-10 am, Thursday 1-2 pm, or by appointment

Required Books: Enumerative Combinatorics, Vol. I and II, by Stanley
Designs, Graphs, Codes, and their Links, by Cameron and van Lint

Prerequisites: Familiarity with permutations and combinations, generating functions, posets, and graphs.

Grades and Policies

The following table summarizes how the course will be graded.

Activity	**Percent of Grade**	**Date**
Homework	35%	Due every other Friday in class
Midterm Exam	20%	March 13
Final exam/Qual	20%	May 13
Final Project	25%	Due May 8; proposal due Feb 14

Homework: will be posted every other Friday and will be due two Fridays from then in class (on paper, or printed out if you typed it in LaTeX). Each homework problem will be assigned a number of points based on Stanley's difficulty ratings, which are listed in the textbook as:

1 - routine, straightforward
2 - somewhat difficult or tricky
3 - difficult
4 - extraordinarily difficult
5 - unsolved

Modifiers of (+) and (-) are used on the rankings as well to differentiate further between difficulties. Therefore a problem ranked 1- is rather trivial, whereas 2+ is a hard graduate-level homework problem.

The number of points you can earn for each rank of problem is as follows:

Rank	1−	1	1+	2−	2	2+	3−	3	3+	4−	4	4+	5
Points	1	1	2	3	3	4	8	9	10	10	10	10	∞

The points you earn are cumulative, and each homework is graded out of a maximum of 10 points. You can choose any problems of the appropriate difficulties in order to score all 10 points. For instance, if you hand in correct solutions to two 2+ level problems and one 1+, that will be a score of 10.

You may hand in a set of problems whose total score is greater than 10 if and only if removing any one of the problems will make the total less than 10. For instance, you may hand in three 2- problems and a 1+, because the total number of points is 11 but removing any one of them will reduce the total to either 8 or 9. But you may not hand in three 2- problems and two 1-, because removing either of the 1- problems will make the total score be 10. **If you hand in an invalid set of problems, you get an automatic zero for that assignment.**

Your score on the homework will be

$$\min(T, 10)$$

where T is the total number of points of the problems you handed in correct solutions to. There is no partial credit; you either get full points or zero points for each problem that you hand in. **Make sure you clearly indicate which problems you are handing in and what their difficulty ratings are!**

Exercise 1. How many different combinations of problem rankings can you hand in so that, if your solutions are all correct, you will get a perfect score?

(Assume order doesn't matter: for instance, a homework that consists of two 2+ level problems and one 1+ is only counted as one possibility, independent of the order in which they appear on your write-up.)

No late homework (defined as later than 9:00 am on the Friday that the homework is due) will be accepted. A missed homework assignment is counted as a zero. In order to account for the possibility of an illness or other unexpected emergency, your lowest homework grade will be dropped when computing your final homework average.

Collaboration is permitted, but as in research, you must list all coauthors on a problem's solution at the top of the page.

Midterm Exam: There will be an in-class midterm exam on March 13.

Final Exam: The final exam will be comprehensive, on all the material covered during the course, on Wednesday, May 13 at 11:50 AM - 1:50 PM on Zoom. This will serve as the qualifying exam as well, for those taking it as a qualifier.

Due to the Coronavirus pandemic, the final exam will be held on ProctorU. You will handwrite your solutions and then scan or photograph and upload them to the ProctorU system.

Final Project: Students must write a paper, between 5 and 15 pages in length (the page number 5 must appear on your document, and the page number 16 must not), on the combinatorial topic of their choice as a final project for this class. You may either work by yourself or with one partner.

If working in a group, you will be asked to briefly indicate which aspects of the project each coauthor contributed to. Assuming everyone contributed a reasonable amount (such as typing a few of the proofs, or reading some of the background materials in depth), all coauthors will receive the same grade as each other on the final project.

The topic must be in combinatorics, and can be any one of the following styles:

1. An expository paper giving an overview of the important open problems and recent results in some specific area of combinatorics.
2. A investigation of a specific open problem, including some recent results and perhaps some of your own observations.
3. An investigation of some combinatorial aspect of your own research interests.

You can choose your own topic, but here are some possible ideas to get you started:

- An investigation of any one of the open problems listed on the homeworks from 501 or the early weeks of 502 that you found interesting.
- An expository paper on complex analytic techniques in generating functions (for instance, the proof of the Ramanujan formula for the partition numbers).
- A historical account of the study of symmetric designs and some of their applications (this is one of the main 502 topics, and has some CSU-specific history due to its applications to agriculture).
- A survey of the many proofs of the hook length formula for enumerating Young tableaux.

Other possible keywords that might help you find a modern combinatorial topic ripe for investigation:

- Polytope theory, for instance the permutahedron, associahedron, and the amplituhedron
- Optimization in graph theory
- Applications of graph theory, for instance to neural networks
- Greedoids (these are a variant of Matroids, which will be covered briefly in this class)
- Pattern avoidance in permutations
- Chip-firing on graphs and its relation to tropical geometry
- Schur positivity of Macdonald polynomials

In general, if there is something not on this list that you have always wanted to understand better and you want to do your project on it, just run it by the professor to get it approved.

Goals and Topics

The goal of this class is to go into more depth in some specialized areas of combinatorics.

Tentative Schedule

- Week 1: (Jan 22, 24) Intro/syllabus, Introduction to symmetric functions - m and e bases
- Week 2: (Jan 27, 29, 31) Symmetric function bases h,p,s, Jacobi-trudi, Newton-Girard identities
- Week 3: (Feb 3, 5, 7) Applications of the Schur functions; hook length formula
- Week 4: (Feb 10, 12, 14) RSK, JDT, and the Littlewood-Richardson rule
- Week 5: (Feb 17, 19, 21) Combinatorial Designs: motivation, examples, Fischer's inequality
- Week 6: (Feb 24, 26, 28) Symmetric designs, finite projective planes as designs
- Week 7: (Mar 2, 4, 6) Finite affine planes and orthogonal Latin squares
- Week 8: (Mar 9, 11, 13) Hadamard matrices and other examples; Midterm
- SPRING BREAK
- Week 9: (Mar 23, 25, 27) Strongly regular graphs
- Week 10: (Mar 30, Apr 1, 3) Error correcting codes, Hamming codes
- Week 11: (Apr 6, 8, 10) Linear and cyclic codes; weight enumerator
- Week 12: (Apr 13, 15, 17) Perfect codes and the Golay code
- Week 13: (Apr 20, 22, 24) Matroids: examples, independent sets, bases
- Week 14: (Apr 27, 29, May 1) Matroids: cycles, greedy algorithm, and applications
- Week 15: (May 4, 6, 8) TBA (possible presentations of final projects)