Math 502: Combinatorics II
Homework 6 - Due Apr 17

Instructions: Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for details.

Since we have moved to an online setting, please either type your solutions or take pictures of your handwritten solutions. Then email them to me at Maria.Gillespie@colostate.edu by April 3.

Problems

1. **(BONUS, +1 point onto any homework or midterm of your choice)** Start writing your final project. To earn this bonus point, start your Overleaf document - see my template at

   https://www.overleaf.com/read/bkkvppggyzww

   to get started - and type at least one page worth of material, whether it consists of background/definitions, an outline, or a proof or two. Share the overleaf document with me by sending me the read link.

2. (2) [3 points] A *character* χ of an abelian group A is a homomorphism from A to the multiplicative group of non-zero complex numbers. The *character table* of A is the matrix whose (i, j) entry is the value of the ith character on the jth element of A (for some ordering of the group elements and the characters). Show that, if A is a direct product of cyclic groups of order 2, then its character table is a Hadamard matrix of Sylvester type.

3. (2) [3 points] Cameron and Van Lint chapter 2 problem 1

4. (1) [1 point] Find the parameters (n, k, λ, μ) for the Petersen graph.

5. (1) [1 point] Show that the Petersen graph is the complement of the triangle graph $T(5)$.

6. (2-) [3 points] Write $L_2(3)$ to denote the strongly regular graph of size 9 resulting from $u = 2$ in the proof of Theorem 2.3 in the book. Write $\overline{T(6)}$ for the strongly regular graph of size 15 resulting from $u = 3$. Show that $L_2(3)$ appears as a vertex-induced subgraph of $\overline{T(6)}$.

7. (1+) [2 points] Finish the proof of the fact that the complement of a strongly regular graph of type (n, k, λ, μ) is strongly regular of type $(n, n-k-1, n-2k+\mu-2, n-2k+\lambda)$, by showing that the parameter $n - 2k + \lambda$ is correct. (We derived the other three parameters in class.)

8. (1+) [2 points] Show that the pentagon graph is a rank three graph.

9. (2+) [4 points] Show that the graph $L_2(3)$ is a rank three graph.

10. (1+) [2 points] Cameron and Van Lint chapter 2 problem 4(ii)

11. (2+) [4 points] Cameron and Van Lint chapter 2 problem 8(i)

12. (3-) [8 points] Cameron and Van Lint chapter 2 problem 13