Math 501 Lecture 1

1. Intros:
   - Names and spellings (on board)
   - Anything you'd like to share about yourself

2. Going over syllabus

3. Exercise 1 from syllabus: How many valid combinations of problem levels can you hand in?

→ Brute force but with careful bookkeeping:

Types	# possibilities	
• 5, 4+, 4, 4−, or 3+	5	18
• (3, x) for x = 1− through 3	8	
• (3−, x) for x = 1− through 3−	5	
• (3−, x, y) for x, y ∈ {1−, 1} decreasing	3	10
• (2+, 2+, x) or (2+, 2+, x, y)	7	
• (2+, 2(−), 2(−)) etc	3	17
• (2+, 2(−), 1+, 1(±))	6	
• (2+, 2(−), x, y, z) x, y, z ∈ {1−, 1}	8	
• (2+, 1+, 1+, 1+) (2+, 1+, 1+, x, y) (2+, 1+, x, y, z, w) (2+, x, y, z, w, s, t)	1+3+5+7 = 16	
• (2(−), 2(−), 2(−), 2(−))	5	
• (2(−), 2(−), 2(−), 1(±))	12 = 4·3	
• (2(−), 2(−), 1+, 1+) x y etc	3·(1+3+5) = 27	

- $(2(-), 1+, 1+, 1+, 1+)$ $\qquad 2(1+2+4+6+8) = 2\cdot 21 = 42$

  $(2(-), 1+, 1+, 1+, x)$

  $(2(-), 1+, 1+, x, y, z)$

  $\vdots$

  $(2(-), x_1, \ldots, x_7)$

- $(1+, 1+, 1+, 1+, 1+)$ $\qquad 1+3+5+\cdots+11 = 36$

  $(1+, 1+, 1+, 1+, x, y)$

  $\vdots$

  $(x_1, \ldots, x_{10})$

$$\underbrace{18+10+17}+\underbrace{16+17}+27+42+36$$

$$= 45 + 33 + 27 + 78$$

$$= 45 + 60 + 78$$

$$= 105 + 78$$

$$= 183$$

$\to$ Method with multiplying polynomials:

Suppose we only have 2+ and 2 level problems (worth 4 points and 3 points respectively). Consider the product

$$(1+x^4+x^8+x^{12}+\cdots)(1+x^3+x^6+x^9+x^{12}+\cdots)$$

A term in this product, expanded, consists of a product of monomials $x^{4k} \cdot x^{3j} = x^{4k+3j}$.

Note that the exponent is the total # points from k (2+)-level problems and j 2-level problems. Thus the number of ways to get a score of n is the coeff of x^n in this product.

(Since combinations must be valid, we can truncate both at x^{12}.)

General setting: let's set aside the 10 pt and ∞ pt problems...

$$P(x) = \underbrace{(1+x^9+x^{18})}_{\text{level 3}} \underbrace{(1+x^8+x^{16})}_{\text{level 3-}} \cdots (1+x^4+x^8+x^{12})(1+x^3+x^6+x^9+x^{12})^2$$
$$(1+x^2+x^4+x^6+x^8+x^{10})$$
$$(1+x+\cdots+x^{10})^2$$

Coeff of x^{10} in $P(x)$: 152 (sage)

$$P'(x) = \frac{P(x)}{(1+x+\cdots+x^{10})^2}$$

Coeff of x^{11} in $P'(x)$: 13

$$P''(x) = P'(x) / (1+x^2+\cdots+x^{10})$$

Coeff of x^{12} in $P''(x)$: 9

$P'''(x) = P''(x)/(1+x^3+x^6+x^9+x^{12})^2$

Coeff of x^{13} in $P'''(x)$: $\underline{1}$

$P''''(x) = P'''/(1+x^4+x^8+x^{12}) = (1+x^9+x^{18})(1+x^8+x^{16})$

Coeffs of x^{14}: 0
x^{15}: 0
x^{16}: 1
x^{17}: 1

$Q = 1+x^9+x^{18}$:
Coeff of x^{18}: 1

Then 5 more from 10pt and ∞ possibilities

<u>Total:</u>

$152+13+9+1+1+1+1+5 = 153+10+20 = 183$