

Math 501: Combinatorics

Homework 1

Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for scoring details.

NOTE: All problems from Stanley are from the second edition, which is available for free on his website here: <http://www-math.mit.edu/~rstan/ec/ec1.pdf>

Problems

1. (2+) [4 points] Let a_n be the number of ways to hand in a subset of the problems on this homework (Homework 1) whose total score is n , *even if handing in such a homework would be against the rules*. Note that different problems of the same rating are now thought of as distinct; for instance, if you hand in parts (a) through (e) of the next problem, that is different from handing in parts (h) through (l), even though they all have difficulty level 1+.
Find a factorization for the generating function for the sequence a_n , and explain how you found your answer. Also, compute a_{10} .
2. (1+) [2 points] Prove that a function is bijective if and only if it has an inverse.
3. (1+) [2 points] Prove that addition and multiplication of cardinalities is well-defined. That is, prove that if there is a bijection $f : A \rightarrow C$ and there is also a bijection $g : B \rightarrow D$, then there are bijections from $A \sqcup B$ to $C \sqcup D$, and from $A \times B$ to $C \times D$.
4. (1+) [2 points] How many paths are there in the plane from $(0, 0)$ to $(6, 6)$ that pass through the point $(3, 3)$, such that each step in the path is of the form $(1, 0)$ or $(0, 1)$ (i.e., unit distance due east or north)?
5. (1+ each) [2 points each] Stanley chapter 1 problem 2, parts (a) through (l). Each part is counted as a separate problem, each considered to be difficulty rating (1+).
6. (2-) [3 points] Stanley chapter 1 problem 3(a)
7. (1+) [2 points] Stanley chapter 1 problem 3(b)
8. (3-) [8 points] Stanley chapter 1 problem 3(c)
9. (3-) [8 points] Stanley chapter 1 problem 3(d)
10. (2-) [3 points] Stanley chapter 1 problem 35(a)
11. (2+) [4 points] Stanley chapter 1 problem 35(f)