Math 501 - Combinatorics
Fall 2019

Basic Information

Instructor: Maria Gillespie, Maria.Gillespie@colostate.edu
Office: 112 Weber Building

Course web page: http://mathematicalgemstones.com/maria/Math501Fall19.php
See also the Canvas course page.

Class time and location: MWF 2:00 pm, E 104 (Engineering Building)

Office hours: Monday 3-4 pm, Thursday 1-2 pm, by appointment, or by knock

Final Exam: Take-home (Due date TBD)

Books: Enumerative Combinatorics, Vol. I and II, by Stanley
(Optional:) Combinatorial Species and Tree-like Structures, by Bergeron, Labelle, and Leroux

Grades and Policies

The following table summarizes how the course will be graded.

Activity	**Percent of Grade**	**Date**
Homework	70%	Due Fridays in class
Final exam	30%	Take-home during finals week

Homework: will be posted each Friday and will be due the following Friday in class (on paper, or printed out if you typed it in LaTeX). Each homework problem will be assigned a number of points based on Stanley's difficulty ratings, which are listed in the textbook as:

1 - routine, straightforward
2 - somewhat difficult or tricky
3 - difficult
4 - extraordinarily difficult
5 - unsolved

Modifiers of (+) and (-) are used on the rankings as well to differentiate further between difficulties. Therefore a problem ranked 1- is rather trivial, whereas 2+ is a hard graduate-level homework problem.

The number of points you can earn for each rank of problem is as follows:

Rank	1−	1	1+	2−	2	2+	3−	3	3+	4−	4	4+	5
Points	1	1	2	3	3	4	8	9	10	10	10	10	∞

The points you earn are cumulative, and each homework is graded out of a maximum of 10 points. You can choose any problems of the appropriate difficulties in order to score all 10 points. For instance, if you hand in correct solutions to two 2+ level problems and one 1+, that will be a score of 10.

You may hand in a set of problems whose total score is greater than 10 if and only if removing any one of the problems will make the total less than 10. For instance, you may hand in three 2- problems and a 1+, because the total number of points is 11 but removing any one of them will reduce the total to either 8 or 9. But you may not hand in three 2- problems and two 1-, because removing either of the 1- problems will make the total score be 10. **If you hand in an invalid set of problems, you get an automatic zero for that assignment.**

Your score on the homework will be

$$\min(T, 10)$$

where T is the total number of points of the problems you handed in correct solutions to. There is no partial credit; you either get full points or zero points for each problem that you hand in. **Make sure you clearly indicate which problems you are handing in and what their difficulty ratings are!**

Exercise 1. How many different combinations of problem rankings can you hand in so that, if your solutions are all correct, you will get a perfect score?

(Assume order doesn't matter: for instance, a homework that consists of two 2+ level problems and one 1+ is only counted as one possibility, independent of the order in which they appear on your write-up.)

No late homework (defined as later than 3:00 pm on the Friday that the homework is due) will be accepted under any circumstances. A missed homework assignment is counted as a zero. In order to account for the possibility of an illness or other unexpected emergency, your lowest homework grade will be dropped when computing your final homework average.

Collaboration is permitted, but as in research, you must list all coauthors on a problem's solution at the top of the page.

Final Exam: The take-home final exam will be comprehensive, on all the material covered during the course. No collaboration is allowed on the final exam.

Goals and Topics

The goal of this class is to give an overview of the wide variety of topics and techniques in both classical and modern combinatorial theory. A tentative schedule of topics is listed below.

Tentative Schedule

- Week 1: (Aug 26, 28, 30) What is combinatorics?; Basics of enumeration
- Week 2: (Sep 4, 6) The Twelvefold Way, Stirling numbers, permutation notations
- Week 3: (Sep. 9, 11, 13) Permutation statistics and q-analogs
- Week 4: (Sep. 16, 18, 20) Formal series, ordinary generating functions, solving recursions
- Week 5: (Sep. 23, 25, 27) Transfer matrix method, Catalan numbers, Exponential generating functions
- Week 6: (Sep. 30, Oct 2, Oct 4) Species and Trees, Cayley's theorem
- Week 7: (Oct 7, 9, 11) Graph theory: Matrix-Tree theorem and generalizations
- Week 8: (Oct 14, 16, 18) Graph theory: Ramsey's theorem, Probabilistic method, Hall's lemma
- Week 9: (Oct 21, 23, 25) Posets; Hasse diagrams; lattices
- Week 10: (Oct 28, 30, Nov 1) Mobius inversion and inclusion-exclusion
- Week 11: (Nov 4, 6, 8) Sign-reversing involutions; Gessel-Viennot
- Week 12: (Nov 11, 13, 15) Partition bijections, involutions, and associated generating functions
- Week 13: (Nov 18, 20, 22) Counting with Symmetry: Group actions, orbit-stabilizer
- Week 14: (Dec 2, 4, 6) Cayley graphs, automorphism groups
- Week 15: (Dec 9, 11, 13) Burnside's lemma and applications, recap