

Math 501: Combinatorics

Homework 12

Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for scoring details.

For any of problems 3 through 7 below, you may use the previous parts without proof.

Problems

1. (1+) [2 points] Let $a_1 = (0, 0)$, $a_2 = (1, 0)$, $b_1 = (2, 3)$, and $b_2 = (4, 3)$. Use the Lindström-Gessel-Viennot lemma to calculate the number of non-intersecting 2-paths in the lattice grid (using only up and right steps) with sources a_1, a_2 and sinks b_1, b_2 . Then, check your answer via a direct counting method. (Coding or casework are both acceptable approaches here.)
2. (2-) [3 points] Let $\sigma(n) = \sum_{d|n} d$ be the sum of the divisors of n . Apply Möbius inversion to the equation above (in the poset of positive integers under divides) and give a direct proof of the resulting inverted equation.

Do the same for the equation $D(n) = \sum_{d|n} 1$ for the quantity $D(n)$, the number of divisors of n .

3. (2-) [3 points] Stanley chapter 2 problem 34(a)
4. (2-) [3 points] Stanley chapter 2 problem 34(b)
5. (1+) [2 points] Stanley chapter 2 problem 34(c)
6. (2) [3 points] Stanley chapter 2 problem 34(d)
7. (2+) [4 points] Stanley chapter 2 problem 34(e)
8. (2) [3 points] Stanley chapter 2 problem 35(b)
9. (3-) [8 points] Stanley chapter 2 problem 37