Math 501: Combinatorics
Homework 11

Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for scoring details.

Problems

1. (1) [1 point] If the rank generating function of a graded poset P factors over $\mathbb{Z}$, can we necessarily write P as a product of two posets having those factors as their generating functions? Why or why not?

2. (2-) [3 points] Draw the Hasse diagrams of all 15 lattices on six elements. Which are upper semimodular? Which are modular? Distributive? Atomic?

3. (2) [3 points] Show that the number of bijective linear extensions of the poset $[2]\times[n]$ to a total ordering on $[2n]$ is given by the Catalan number C_n.

4. (2) [3 points] Stanley chapter 3 problem 13(b).

5. (2) [3 points] Stanley chapter 3 problem 122.

6. (1+) [2 points] Describe a recursion for the square of the Möbius function, μ^2, in the incidence algebra of a poset, by thinking of it as the inverse function of ζ^2.

7. (2+) [4 points] Show that a finite graded lattice L is modular (as defined by the rank condition given in class) if and only if for all $x, y, z \in L$ with $x \leq z$, we have $x \vee (y \wedge z) = (x \vee y) \wedge z$.

8. (5) [∞ points] Stanley chapter 3 problem 135(b).

9. (3) [9 points] Stanley chapter 3 problem 136.

10. Let P be a poset in which every interval $[x, y]$ is finite. Show that, in the incidence algebra $I(P)$:
    - (1+) [2 points] f is invertible if and only if $f(x, x) \neq 0$ for all x,
    - (2-) [3 points] $fg = \delta$ if and only if $gf = \delta$ (inverses are two-sided),
    - (1) [1 point] If f is invertible then its inverse f^{-1} is unique.