

Math 567: Abstract Algebra I

Homework 4

10 points total. Due Friday, Feb 19 by 11:10 am in class.

Problems

1. (2 points) Suppose $N_1 \subseteq N_2 \subseteq N_3 \subseteq \dots$ is a weakly increasing (possibly strictly increasing) chain of submodules of an R -module M . Show that $\bigcup_i N_i$ is also a submodule of M .
2. (2 points) Artin problem 9.2
3. (1 point each) Artin problems 9.1(a), (b), (c)
4. (1 point) I'm thinking of a whole number between 1 and 100. When divided by 4 or 3, it has a remainder of 1, and when divided by 5, it has a remainder of 4. What is my number?
5. (2 points) I'm thinking of a polynomial in x . When I divide it by $x - 1$ I get a remainder of 3. When I divide it by $x + 1$ I get a remainder of -1 . If I were to divide it by $x^2 - 1$, what would its remainder be?

Bonus Problem

(+1 point:) Read through the first 17 pages of this bedtime story by Ravi Vakil, and answer the Homework question on Page 17 of the pdf about commuting triangles.