Math 567: Abstract Algebra I
Homework 14

10 points total. Due Friday, May 6 by 1:10 pm in class. These are all problems that serve as review for the final; at least one of the problems on the final will be taken verbatim from this list! Many others will be similar in nature.

Problems

1. (1 point) Write down the definitions of R-module, G-module, field extension, and Galois group.
2. (1 point - this was on a previous homework) Show that if an abelian group has the structure of a $\mathbb{Q}$-module, then it has a unique such structure.
3. (1 point - this was on a previous homework) Show that $C_a \oplus C_b = C_{ab}$ for a, b relatively prime using presentation matrices for modules.
4. (1 point) Artin chapter 14 problem 7.3(d)
5. (1 point) Explain why a matrix representation of a finite group G over $\mathbb{C}$ is the same thing as a G-action by linear operators on a complex vector space V. Then explain why the latter is the same thing as a module over the group ring $\mathbb{C}G$.
6. (1 point) Compute the character tables of the Klein four group, the dihedral groups D_4 and D_6, and the cyclic group C_5.
7. (1 point) Compute the character table of the symmetric group S_3, and describe the representation corresponding to each row. Then decompose the representation of S_3 acting on the degree 2 homogeneous part of $\mathbb{C}[x_1, x_2, x_3]$ into irreducibles.
8. (1 point) Prove that any field extension K of F can be thought of as a vector space over F. Also write down the definition of the index $[K : F]$.
9. (1 point) Rationalize the denominator of $\frac{1}{2-3\sqrt[3]{2}+\sqrt[3]{4}}$.
10. (1 point) Give an example of a Galois extension with Galois group isomorphic to S_3, and write down the intermediate fields. Which intermediate fields are Galois extensions over the ground field?