Math 502 - Combinatorics II
Spring 2024

Basic Information

Instructor: Maria Gillespie, Maria.Gillespie@colostate.edu
Office: Weber 125

Course web page: http://mathematicalgemstones.com/maria/Math502Spring24.html
See also the Canvas course page.

Class time and location: MWF 11:00 AM, Weber 15 (in the basement!)

Office hours: TBD - See Canvas announcements

Books: All books are optional for this class. The lectures themselves will serve as the course notes. Some that may be helpful are:

- Enumerative Combinatorics, Vol. II, by Stanley
- Young Tableaux, by Fulton
- Matroid Theory, by Oxley
- Designs, Graphs, Codes, and their Links, by Cameron and van Lint

Grades and Policies

The following table summarizes how the course will be graded.

Activity	**Percent of Grade**	**Date**
Homework	70%	Due Fridays at 11 am
Project Draft	10%	Friday, April 19
Project Paper	10%	Friday, May 3
Presentation	10%	Last two weeks of class

Homework: will be posted each Friday and will be due the following Friday by class time (typed or photographed and uploaded on Canvas). Each homework problem will be assigned a number of points based on my own difficulty ratings, which follow Stanley's rubric:

1 - routine, straightforward
2 - somewhat difficult or tricky
3 - difficult
4 - extraordinarily difficult
5 - unsolved

Modifiers of (+) and (-) are used on the rankings as well to differentiate further between difficulties. Therefore a problem ranked 1- is rather trivial, whereas 2+ is a hard graduate-level homework problem.

The number of points you can earn for each rank of problem is as follows:

Rank	1−	1	1+	2−	2	2+	3−	3	3+	4−	4	4+	5
Points	1	1	2	3	3	4	8	9	10	10	10	10	∞

The points you earn are cumulative, and each homework is graded out of a maximum of 10 points. You can choose any problems of the appropriate difficulties in order to score all 10 points. For instance, if you hand in correct solutions to two 2+ level problems and one 1+, that will be a score of 10.

You may hand in a set of problems whose total score is greater than 10 if and only if removing any one of the problems will make the total less than 10. For instance, you may hand in three 2- problems and a 1+, because the total number of points is 11 but removing any one of them will reduce the total to either 8 or 9. But you may not hand in three 2- problems and two 1-, because removing either of the 1- problems will make the total score be 10. **If you hand in an invalid set of problems, you get an automatic zero for that assignment.**

Your score on the homework will be

$$\min(T, 10)$$

where T is the total number of points of the problems you handed in correct solutions to. There will be partial credit; do your best. **Make sure you clearly indicate which problems you are handing in and what their difficulty ratings are!**

Online format: For this class, I'll be switching to online file uploads on Canvas for handing in homework, rather than on paper in class.

Late homework policy: You may hand in homework late, but one point will be deducted for each day it is late. In particular, if it is handed in between 11:10 am on the Friday it is due and 10:59:59 pm on Saturday, it is counted as one day late. If it is handed in between 11:00 on Saturday and 10:59:59 pm on Sunday, it is counted as two days late, and so on.

These deductions will bottom out at -10 points, and if the total score is negative then the official homework grade is simply a 0.

Collaboration: on homeworks is permitted, but as in research, you must list all coauthors on a problem's solution at the top of the page. Your writing must also be your own; if I see strong evidence of copying solutions word for word from each other, the assignment will be marked as a zero.

Project: There will be no exams in this course; instead, you will do a project exploring some area of the class in more depth. See below for some project ideas. You may work independently if you wish, but I **highly encourage** you to form groups of size 2 or 3. The project must consist of at least 4 typed pages in LaTeX, with 1 inch margins and 11 point font, single spaced, that explores a combinatorial topic in depth. It could be on an open problem in combinatorics that you summarize the published literature on, or could simply consist of an exposition of a proof of a result that we did not have time to prove in class. There is no upper limit on the number of pages, but you will be graded partly on style and readability as well as accuracy, so lots of poorly written pages does not necessarily translate into a better grade.

Project rough drafts will be due two weeks before the due date so that students can get feedback on their writing before the final project is due.

If you are just getting started with LaTeX, a good place to start is the website Overleaf, which allows you to create LaTeX documents on the cloud without having to download and install LaTeX on your computer or laptop. There will be a template provided for you to get started with as well.

Presentations: In the last week of class, each project group will have 20 minutes to give a presentation on their project (2 presentations per class period). It may be either a chalk or slide presentation, and if multiple people teamed up on a project, the time speaking should be split between the people with each person speaking for at least 5 minutes.

Goals and Topics

The goal of this class is to give an overview of the wide variety of topics and techniques in both classical and modern combinatorial theory. A tentative schedule of topics is listed below.

Tentative Schedule

- Week 1: (Jan 17, 19) Intro, review of symmetric functions

- Week 2: (Jan 22, 24, 26) Antisymmetric functions and Schurs, omega involution
- Week 3: (Jan 29, 31, Feb 2) Hall inner product, Omega function
- Week 4: (Feb 5, 7, 9) Young tableaux - JDT, RSK, Knuth equivalence
- Week 5: (Feb 12, 14, 16) Littlewood-Richardson rule(s)
- Week 6: (Feb 19, 21, 23) Crash course on representation theory and group actions
- Week 7: (Feb 26, 28, Mar 1) Schur functions and S_n representations, Murnaghan-Nakayama rule
- Week 8: (Mar 4, 6, 8) Orbit-stabilizer, Burnside
- SPRING BREAK
- Week 9: (Mar 18, 20, 22) Designs
- Week 10: (Mar 25, 27, 29) Matroids - theory
- Week 11: (Apr 1, 3, 5) Matroids - applications (project topics due)
- Week 12: (Apr 8, 10, 12) Finite geometries
- Week 13: (Apr 15, 17, 19) Grassmannians, finite and infinite (project drafts due)
- Week 14: (Apr 22, 24, 26) Connection back to Schur functions
- Week 15: (Apr 29, May 1, 3) *Group project presentations* (20 min each)

Final project suggestions

- Proof(s) of the hook length formula
- The Stanley-Stembridge conjecture
- The Shuffle and Delta conjectures
- The Macdonald positivity conjecture
- Kronecker coefficients and the problem of computing them efficiently
- Knutson-Tao's proof of the Saturation conjecture, and related problems that remain open
- The S_n action on the coinvariant ring
- Flag varieties, their cohomology, and the coinvariant ring
- Links between matroids and hyperplane arragments
- A project of your own Design

Classroom environment

In order to help make our classroom an excellent place to be in and learn mathematics, please keep in mind the following principles:

- *Speak up in class!* If you don't understand something, no matter how small, chances are someone else in the class doesn't understand it either, and asking will help me address the confusion and make things clearer. Off-the-wall ideas and comments are also always encouraged.
- *Talk to each other!* If I give a problem for you all to think about during class, chatting with your neighbor is allowed and encouraged; explaining helps both the explainer and the listener to understand better.
- *Kindness:* The students in this class will be coming from many different backgrounds, both mathematically and as human beings. Please be respectful and encouraging towards each other.

Academic Integrity

This course will adhere to the CSU Academic Integrity Policy as found on the Student Responsibilities page of the CSU General Catalog and in the Student Conduct Code. At a minimum, violations will result in a grading penalty in this course and a report to the Office of Student Resolution Center.

Disabilities

Colorado State University is committed to providing reasonable accommodations for all persons with disabilities. Students with disabilities who need accommodations must first contact the Student Disability Center (SDC) before requesting accommodations for this class.