Math 501 - Combinatorics
Fall 2023

Basic Information

Instructor: Maria Gillespie, Maria.Gillespie@colostate.edu
Office: Weber 125

Course web page: http://mathematicalgemstones.com/maria/Math501Fall23.php
See also the Canvas course page.

Class time and location: MWF 2:00 pm, Engineering Building room 104

Office hours: Monday 3-4 PM, Thursday 2-3 PM

Books:

- Combinatorics: The Art of Counting, by Sagan
- Enumerative Combinatorics, Vol. I and II, by Stanley
- (Optional:) Combinatorial Species and Tree-like Structures, by Bergeron, Labelle, and Leroux

Grades and Policies

The following table summarizes how the course will be graded.

Activity	**Percent of Grade**	**Date**
Homework	70%	Due Fridays in class
Midterm Exam	10%	Fri, Oct 6 in class
Final Exam	20%	Thurs, Dec 14, 11:50 AM - 1:50 pm

Homework: will be posted each Friday and will be due the following Friday in class (on paper, or printed out if you typed it in LaTeX, or by email if you are out sick or hand it in late). Each homework problem will be assigned a number of points based on Stanley's difficulty ratings, which are listed in the textbook as:
1 - routine, straightforward
2 - somewhat difficult or tricky
3 - difficult
4 - extraordinarily difficult
5 - unsolved

Modifiers of (+) and (-) are used on the rankings as well to differentiate further between difficulties. Therefore a problem ranked 1- is rather trivial, whereas 2+ is a hard graduate-level homework problem.

The number of points you can earn for each rank of problem is as follows:

Rank	1−	1	1+	2−	2	2+	3−	3	3+	4−	4	4+	5
Points	1	1	2	3	3	4	8	9	10	10	10	10	∞

The points you earn are cumulative, and each homework is graded out of a maximum of 10 points. You can choose any problems of the appropriate difficulties in order to score all 10 points. For instance, if you hand in correct solutions to two 2+ level problems and one 1+, that will be a score of 10.

You may hand in a set of problems whose total score is greater than 10 if and only if removing any one of the problems will make the total less than 10. For instance, you may hand in three 2- problems and a 1+, because the total number of points is 11 but removing any one of them will reduce the total to either 8 or 9. But you may not hand in three 2- problems and two 1-, because removing either of the 1- problems will make the total score be 10. **If you hand in an invalid set of problems, you get an automatic zero for that assignment.**

Your score on the homework will be

$$\min(T, 10)$$

where T is the total number of points of the problems you handed in correct solutions to. There may be partial credit given on harder problems. **Make sure you clearly indicate which problems you are handing in and what their difficulty ratings are!**

Exercise 1. How many different combinations of problem rankings can you hand in so that, if your solutions are all correct, you will get a perfect score?

(Assume order doesn't matter: for instance, a homework that consists of two 2+ level problems and one 1+ is only counted as one possibility, independent of the order in which they appear on your write-up.)

Late homework policy: You may hand in homework late, but one point will be deducted for each day it is late. In particular, if it is handed in between 2:10 pm on the Friday it is due and 2:09:59 pm on Saturday, it is counted as one day late. If it is handed in between 2:10 on Saturday and 2:09:59 pm on Sunday, it is counted as two days late, and so on.

These deductions will bottom out at -10 points, and if the total score is negative then the official homework grade is simply a 0.

Collaboration: on homeworks is permitted and encouraged. As in research, you must list all coauthors on a problem's solution at the top of the page, and any references at the bottom. Your writing must also be your own; if I see strong evidence of copying solutions word-for-word from each other or from a textbook solution, the assignment will be marked as a zero.

Exams: The final exam will be comprehensive and act as the qualifying exam for this course as well. The midterm will be on October 8 and will give a sense of the style of the final exam.

Goals and Topics

The goal of this class is to give an overview of the wide variety of topics and techniques in both classical and modern combinatorial theory. A tentative schedule of topics is listed below.

Tentative Schedule

- Week 1: (Aug 21, 23, 25) What is combinatorics?; Basics of enumeration
- Week 2: (Aug 28, 30, Sep 1) The Twelvefold Way, Stirling numbers, permutation notations
- Week 3: (Sep. 6, 8) Permutation statistics and q-analogs
- Week 4: (Sep. 11, 13, 15) Formal series, ordinary generating functions, solving recursions
- Week 5: (Sep. 18, 20, 22) Catalan numbers, Exponential generating functions
- Week 6: (Sep. 25, 27, 29) Combinatorial Species I
- Week 7: (Oct 3, 5, 7) Combinatorial Species II, Midterm (Oct 7)
- Week 8: (Oct 9, 11, 13) Graph theory: Matrix-Tree theorem and generalizations
- Week 9: (Oct 17, 19, 20) Graph theory: Markov chains and spanning trees, Ramsey's theorem, Hall's lemma
- Week 10: (Oct 23, 25, 27) Posets; Hasse diagrams; lattices
- Week 11: (Oct 30, Nov 1, 3) Mobius inversion and inclusion-exclusion

- Week 12: (Nov 6, 8, 10) Sign-reversing involutions; Gessel-Viennot
- Week 13: (Nov 13, 15, 17) Partition bijections, involutions, and associated generating functions
- Week 14: (Nov 27, 29, Dec 1) Symmetric Functions I
- Week 15: (Dec 4, 6, 8) Symmetric Functions II

Topics for Math 502, Spring 2023: Counting with symmetry (orbit-stabilizer and Burnside's Lemma), finite geometries, combinatorial designs, matroids, Young tableaux and the plactic monoid, representation theory of the symmetric group, coinvariant rings

Classroom environment

In order to help make our classroom an excellent place to be in and learn mathematics, please keep in mind the following principles:

- ***Speaking up in class is encouraged!*** If you don't understand something, no matter how small, chances are someone else in the class doesn't understand it either, and asking will help me address the confusion and make things clearer. Off-the-wall ideas and comments are also always encouraged.
- ***Talk to each other!*** If I give a problem for you all to think about during class, chatting with your neighbor is encouraged; explaining helps both the explainer and the listener to understand better.
- ***Kindness:*** The students in this class will be coming from many different backgrounds, both mathematically and as human beings. Please be respectful and encouraging towards each other.

Academic Integrity

This course will adhere to the CSU Academic Integrity Policy as found on the Student Responsibilities page of the CSU General Catalog and in the Student Conduct Code. At a minimum, violations will result in a grading penalty in this course and a report to the Office of Student Resolution Center.

Disabilities

This is a disability-inclusive classroom. Students with disabilities who need accommodations can ask me directly, though I may ask you to contact the Student Disability Center (SDC) before approving accommodations such as extra time on exams. However, if there are other accommodations that are not related to course/grading policy that could help you participate more fully in class, please let me know.